1974 Q10 DE

Forces

↑ $-0.02v$

↓ $\cdot 1g$

$x = 0$, $t = 0$, $v = 0$

typical point: $x = X$, $t = t$, $v = v$.

Signs

↓ +

N II $\Rightarrow$ $F = ma$

$\Rightarrow \cdot 1g - 0.02v = 0.1\frac{dv}{dt}$

$\Rightarrow 98 - 2v = 10\frac{dv}{dt}$

$\Rightarrow 49 - v = 5\frac{dv}{dt}$.

$\Rightarrow \int \frac{1}{5}\,dt = \int \frac{dv}{49-v}$

Now $\int \frac{dv}{49-v}$

Let $u = 49 - v$

$du = -dv$

$\therefore \int \frac{dv}{49-v} = -\int \frac{du}{u}$

$= -\ln u$

$= -\ln(49-v)$

$\Rightarrow \frac{t}{5} + \text{Const} = -\ln(49-v)$

$t = 0,\ v = 0 \Rightarrow$

Const $= -\ln 49$.

$\therefore \frac{t}{5} - \ln 49 = -\ln(49-v)$

$\Rightarrow \frac{t}{5} = -\ln(49-v) + \ln 49$

$\Rightarrow -\frac{t}{5} = \ln(49-v) - \ln 49$

$\Rightarrow -\frac{t}{5} = \ln\left[\frac{49-v}{49}\right]$ $\Rightarrow e^{-\frac{t}{5}} = \frac{49-v}{49}$

$\Rightarrow 49e^{-\frac{t}{5}} = 49 - v$.

$\Rightarrow$ ~~$v = 49[49$~~

$\Rightarrow v = 49\left[1 - e^{-\frac{t}{5}}\right]$.

$\lim_{t \to \infty} e^{-\frac{t}{5}} = 0$ so $\lim_{t \to \infty} v = 49[1 - 0]$

$= 49$.

Find distance travelled

$v = 49\left[1 - e^{-\frac{t}{5}}\right] \Rightarrow \frac{dx}{dt} = 49\left[1 - e^{-\frac{t}{5}}\right]$

$\Rightarrow \int dx = \int 49\left[1 - e^{-\frac{t}{5}}\right]dt + \text{Const}$

$\Rightarrow x = 49t + 5e^{-\frac{t}{5}} + \text{Const}$.

$v = 0,\ t = 0$, when $x = 0$ $\Rightarrow 0 = 0 + 5e^{0} + \text{Const}$.

$\Rightarrow \text{const} = -5$.

$\therefore$ distance travelled in time t, $x = 49t + 5e^{-\frac{t}{5}} + 5$.